

Research Methodology Series

Mediator and Moderator Variables in Social Science Research

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Why Studying Moderators or Mediators

- Mediators elucidate the *mechanism* behind the observed relationship between X and Y, thus can help advance theory.
- Mediators can help identify the *effective components* of an intervention, thus can further help develop cost-effective interventions.
- Moderators indicate *under what conditions* or *to whom* the relationship between X and Y exist.
 - Theory advancing
 - Intervention selection

Frameworks Discussed Today

- Barron and Kenny (1986)
- Kraemer, Wilson , Fairburn and Agras (2002)

What is a Moderator?

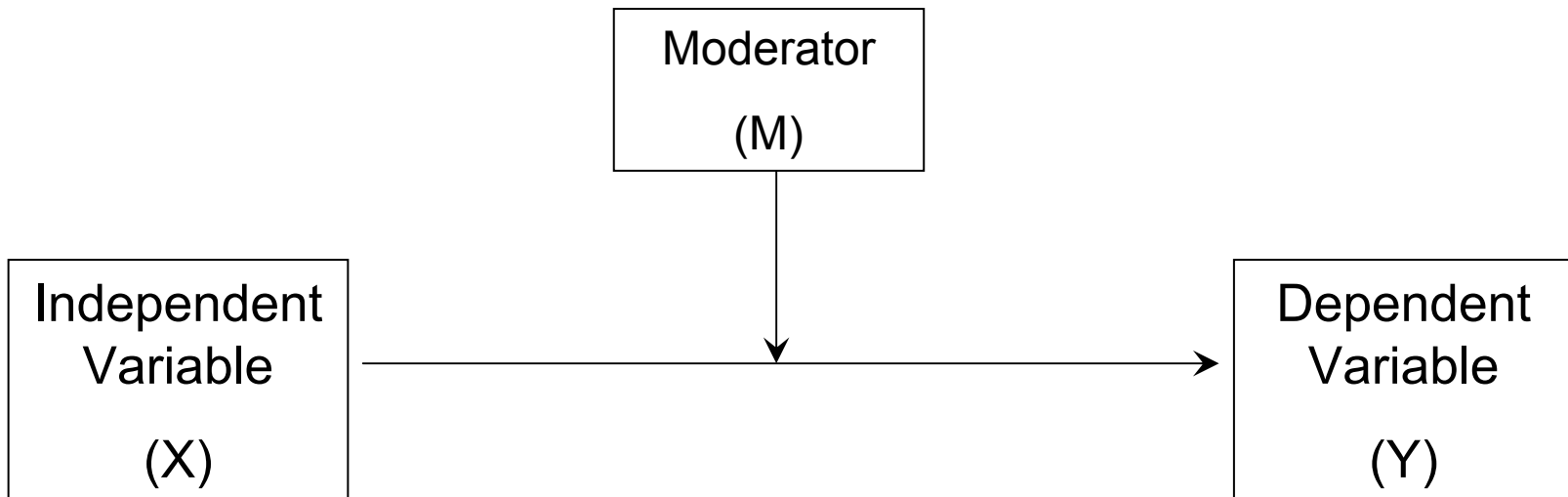
- Baron and Kenny (1986)

- “a moderator is a qualitative (e.g., sex, race, class) or quantitative (e.g., level of reward) variable that affects the direction and/or strength of the relation between an independent or predictor variable and a dependent or criterion variable.” (p.1174)
- relationship between two variables changes as a function of the moderator variable” (p.1174)

- Relationship between variables X and Y depends on the level of M.

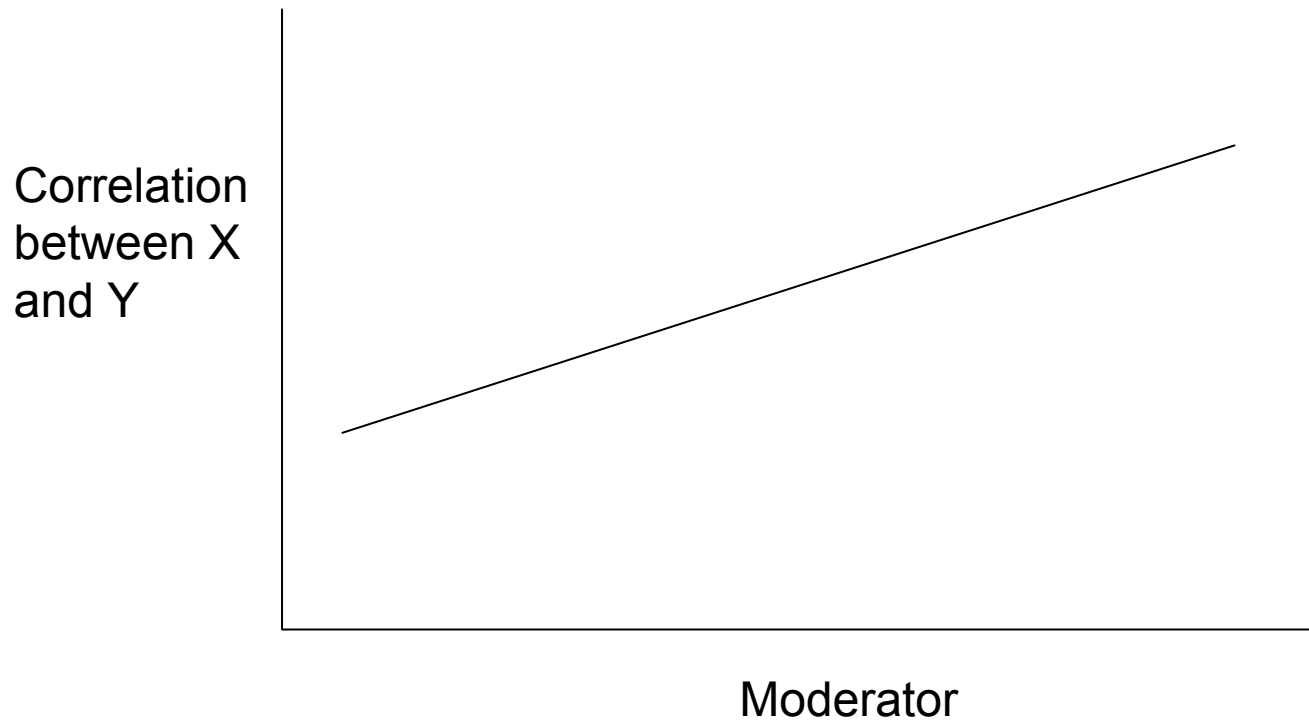
- moderator effect = interaction effect

Conceptual Diagram for Moderation



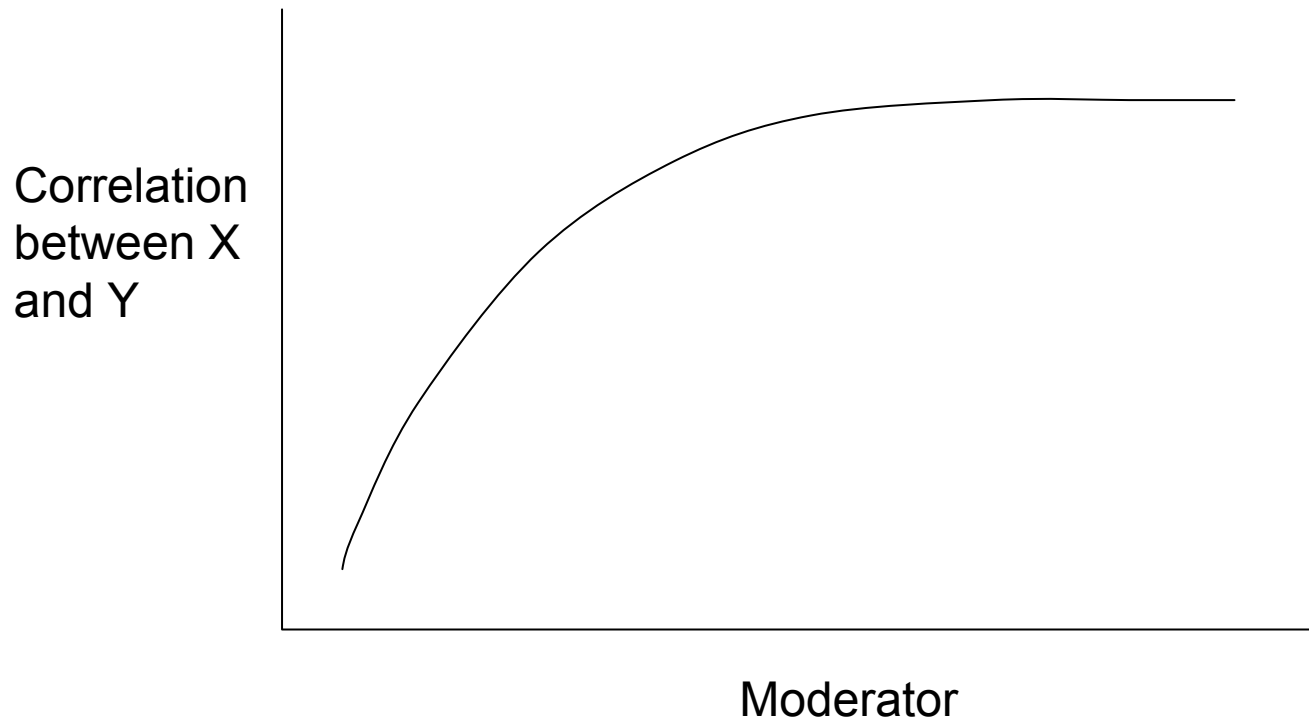
Form of Moderation

■ Linear



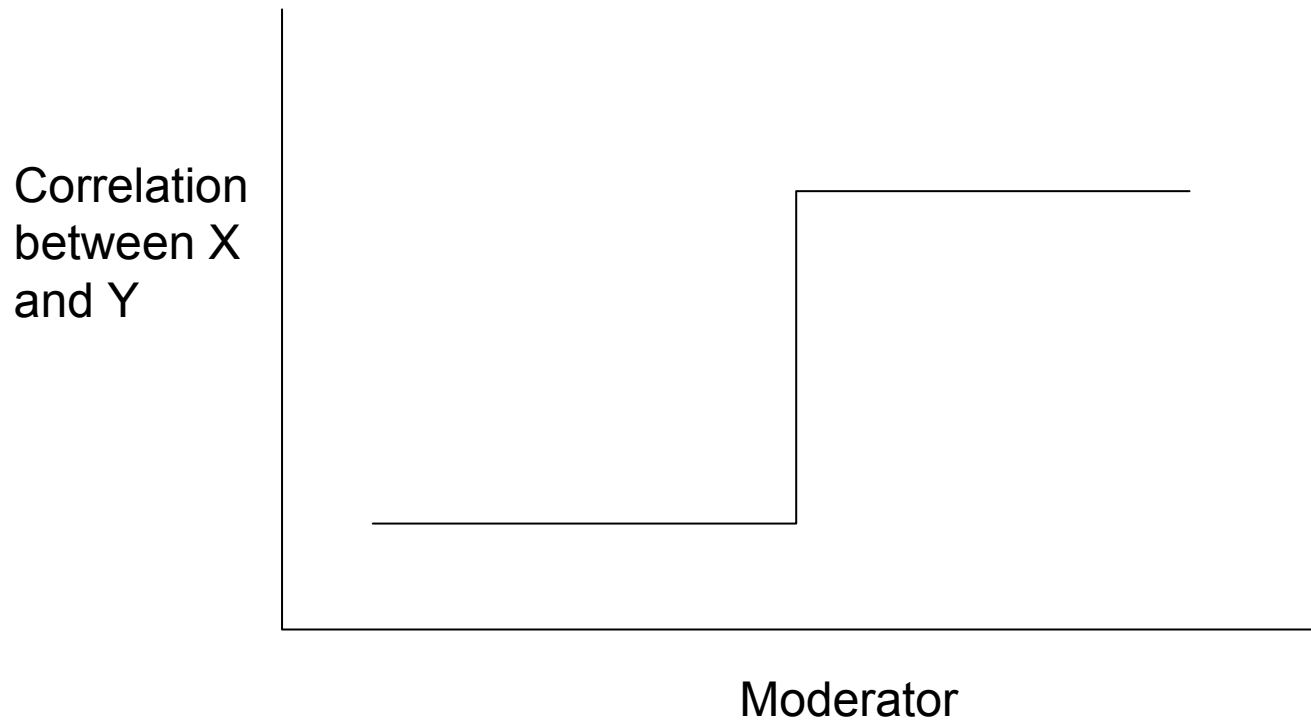
Form of Moderation

■ Nonlinear



Form of Moderation

- Step function



Analysis Approach for Moderator

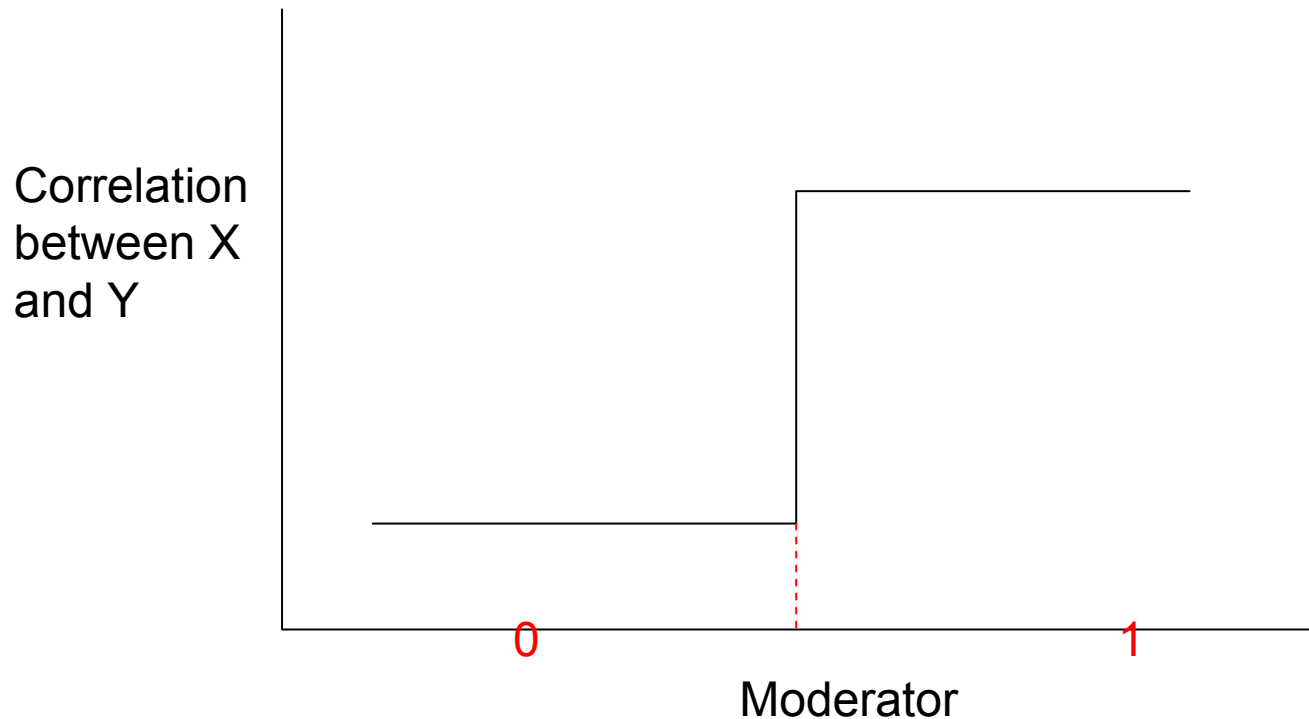
- Moderated multiple regression analysis
 - regressing Y on X, M, and XM simultaneously

$$\hat{Y} = b_0 + b_1X + b_2M + b_3(XM)$$

- moderator effects are indicated by significant effect of XM (i.e., b_3) while X and M are controlled

Form of Moderation

- When moderation exhibit a step function, categorizing M can be useful.



Other Issues related to Detecting Moderator

■ Centering

- Facilitate interpretation
- Mitigate multicollinearity problem

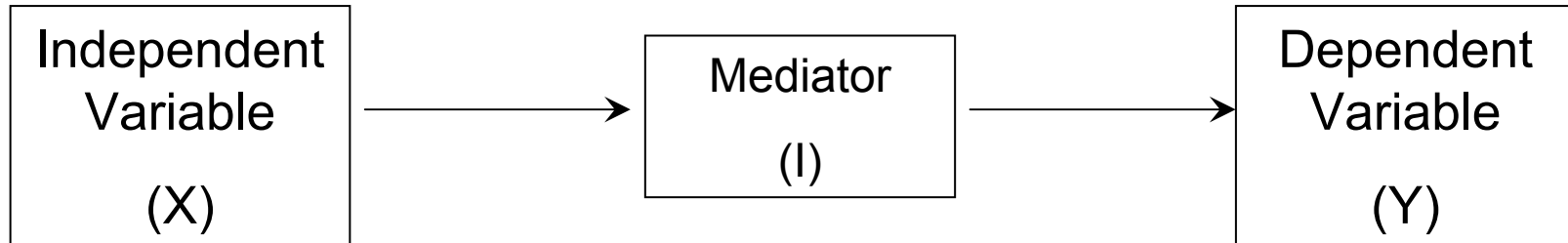
■ Low power for detecting moderation in nonexperimental research (McClelland and Judd, 1993)

- Increase sample size
- Over sample

What is a Mediator

- “mediator...accounts for the relationship between the predictor and the criterion” (p. 1176, Baron & Kenny, 1986)
- “An intervening variable (mediator) transmits the effect of an independent variable to a dependent variable.” (p. 83, MacKinnon, Lockwood, Hoffman, West & Sheets, 2002)
- a.k.a. Intervening Variable

Conceptual Diagram for Mediator



Causal Steps Approach

- Baron & Kenny (1986) analytical approach for mediation

(1) X must affect Y (*regressing Y on X*) $\hat{Y} = \beta_o + \tau X$

(2) X must affect I (*regressing I on X*) $\hat{I} = \beta_o' + \alpha X$

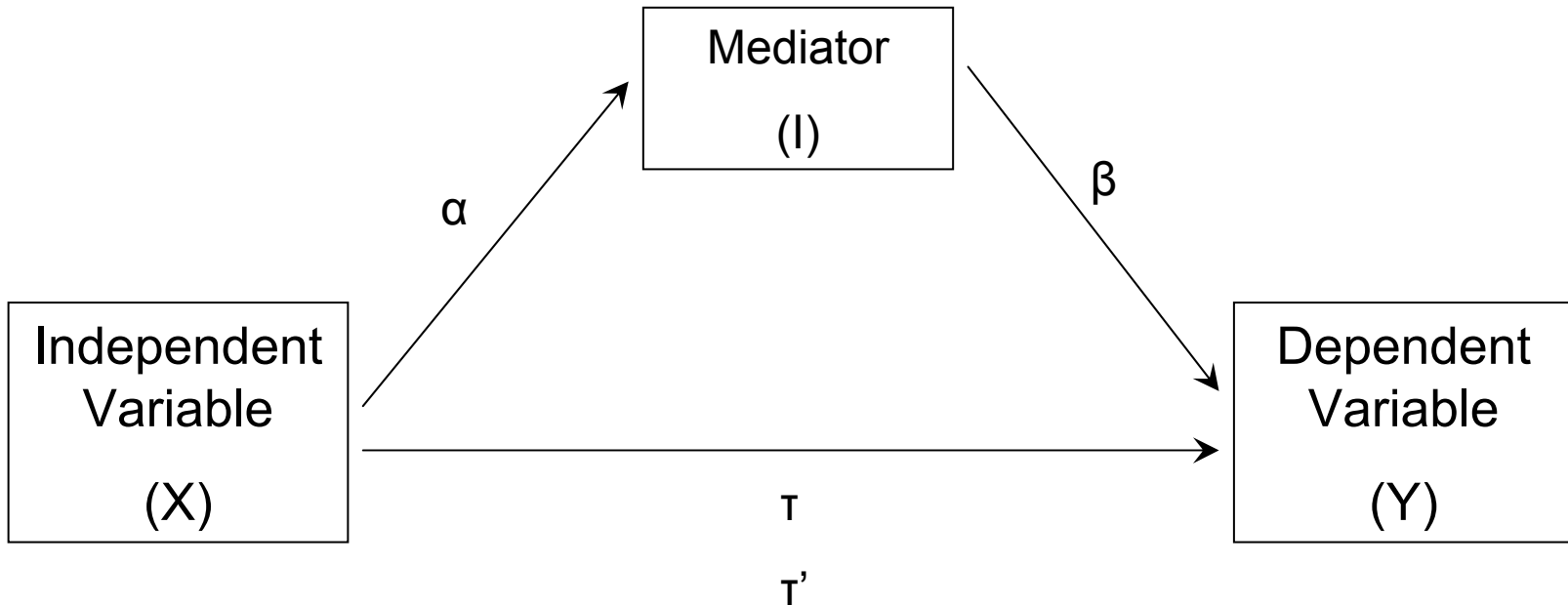
(3) I must affect Y when X is controlled

(regressing Y on X and I simultaneously)

$$\hat{Y} = \beta_o'' + \tau' X + \beta I$$

(4) the effect of X on Y must be less in (3) than in (1)

Conceptual Diagram for Mediator



Indirect Effect

- MacKinnon, Warsi & Dwyer (1995): The τ - τ' test and the $\alpha\beta$ test are algebraically equivalent

- Sobel test (1982) for indirect effects ($\alpha\beta$)

$$s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2}$$

- Compare the critical ratio (ab/s_{ab}) to appropriate critical value from the Z distribution for a given significant level.

Variations of Sobel test

- There are variations for Sobel test

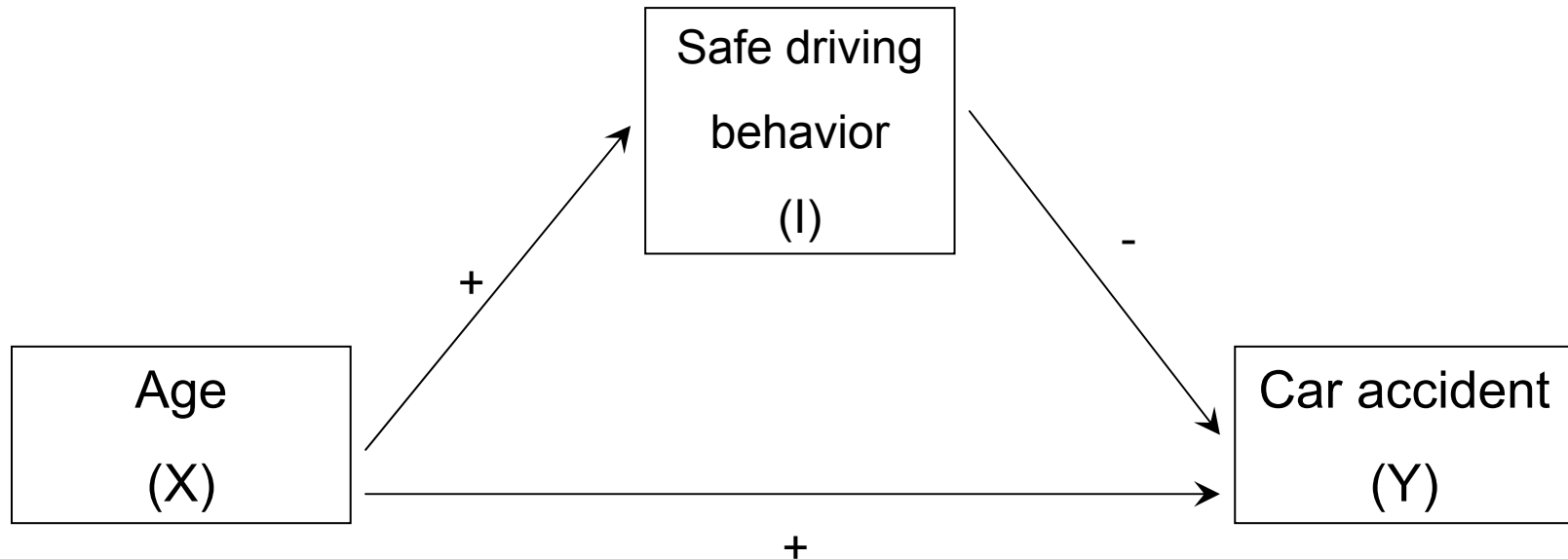
- Goodman (1960): $s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2 - s_a^2 s_b^2}$

- Aroian (1944): $s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2}$

- <http://www.unc.edu/~preacher/sobel/sobel.htm>

Mediation = Indirect effect?

- Consistent intervening
- Inconsistent intervening



Multicollinearity in Mediation

- X and I will be correlated
- Effective sample size is approximately $N(1-r^2)$, where r is the correlation between X and I.

Measurement Errors in Mediators

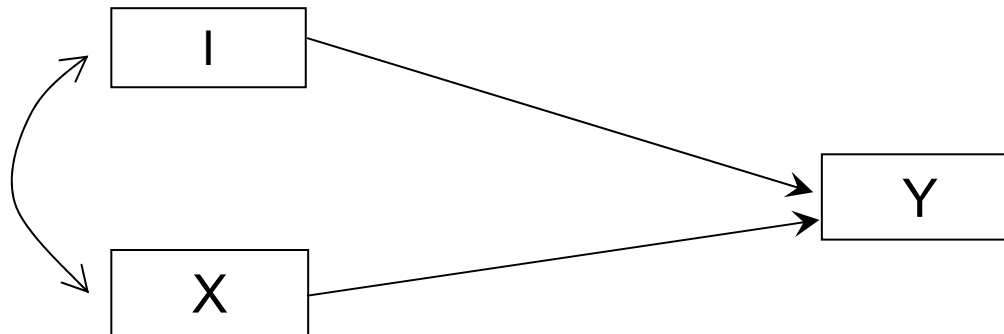
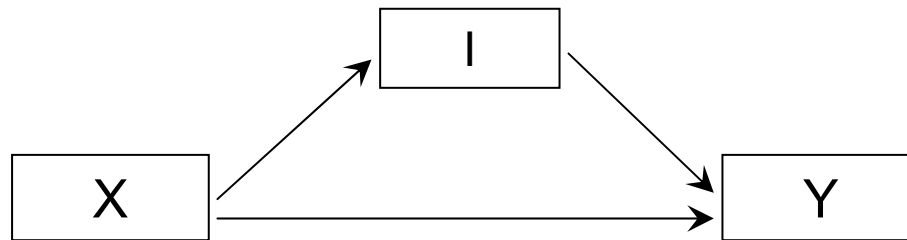
- Using multiple regression for detecting mediation assumes M is measured without errors.
- When measurement error exists but not acknowledged, regression approach leads to underestimation of mediation effect.
- Solution: SEM approach with multiple indicators

Mediation model = Causal model?

- Causal relationship (Shadish, Cook & Campbell, 2002)
 - The cause preceded the effect
 - The cause was related to the effect
 - No plausible alternative explanation for the effect other than the cause

Mediation v.s. confounding effects

- Statistically equivalent models



Distinguish Mediation and Confounding

- Temporal precedence
 - Randomized study
 - Nature of the variable studied
 - Theory
-
- See MacKinnon, Krull & Lockwood (2000) for detail discussions

Kraemer, Wilson , Fairburn and Agras (2002)

- Redefine mediators and moderators in a randomized clinical trials
- Definitions are conceptually similar to Baron and Kenny 1986
- “Treatment moderators specify for whom or under what conditions the treatment works.” (p.878)
- “Treatment mediators identify possible mechanisms through which a treatment might achieve its effects.” (p.878)

Kraemer, Wilson , Fairburn and Agras (2002)

- Emphasizing on temporal order of variables
- Endorse a different analytical approach.
- Using regression model

$$\hat{Y} = b_0 + b_1X + b_2M + b_3(XM)$$

for both mediator and moderator

Moderator in Kraemer et. al. (2002)

- Moderator must be a baseline or pre-randomization characteristics.
- Moderator is uncorrelated with treatment.
- Moderator has an interactive effect with treatment on the outcome.

Mediator in Kraemer et. al. (2002)

- Mediator is a measure of event or change occurring during treatment.
- Mediator must correlate with treatment
- Mediator has either a main or an interactive effect on the outcome

Kraemer, Wilson , Fairburn and Agras (2002)

- See handout

Mediators and Moderators in Research Design

- Direct manipulation of mediators in an experimental design to establish causality.
- Use moderators as stratified variables in sampling for following research
- Moderator effects might suggest potential mediators to be tested
 - Instruction XYZ worked better for low SES students than for high SES students. Why?