Research Methodology Series

Mediator and Moderator Variables in Social Science Research

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Why Studying Moderators or Mediators

- Mediators elucidate the *mechanism* behind the observed relationship between X and Y, thus can help advance theory.

- Mediators can help identify the *effective components* of an intervention, thus can further help develop cost-effective interventions.

- Moderators indicate *under what conditions* or *to whom* the relationship between X and Y exist.
  
  - Theory advancing
  - Intervention selection
Frameworks Discussed Today

- Barron and Kenny (1986)
- Kraemer, Wilson, Fairburn and Agras (2002)
What is a Moderator?

- Baron and Kenny (1986)
  - “a moderator is a qualitative (e.g., sex, race, class) or quantitative (e.g., level of reward) variable that affects the direction and/or strength of the relation between an independent or predictor variable and a dependent or criterion variable.” (p.1174)

- relationship between two variables changes as a function of the moderator variable” (p.1174)

- Relationship between variables X and Y depends on the level of M.

- moderator effect = interaction effect
Conceptual Diagram for Moderation

Independent Variable (X) → Moderator (M) → Dependent Variable (Y)
Form of Moderation

- Linear

Correlation between X and Y

Moderator
Form of Moderation

- Nonlinear

Correlation between X and Y

Moderator
Form of Moderation

- Step function

Correlation between X and Y

Moderator
Analysis Approach for Moderator

- Moderated multiple regression analysis
  - regressing Y on X, M, and XM simultaneously
    \[ \hat{Y} = b_0 + b_1 X + b_2 M + b_3 (XM) \]
  - moderator effects are indicated by significant effect of XM (i.e., \( b_3 \)) while X and M are controlled
Form of Moderation

- When moderation exhibit a step function, categorizing M can be useful.

Diagram:
- Correlation between X and Y
- Graph with x-axis labeled Moderator, ranging from 0 to 1.
Other Issues related to Detecting Moderator

- Centering
  - Facilitate interpretation
  - Mitigate multicollinearity problem

- Low power for detecting moderation in nonexperimental research (McClelland and Judd, 1993)
  - Increase sample size
  - Over sample
What is a Mediator

- “mediator...accounts for the relationship between the predictor and the criterion” (p. 1176, Baron & Kenny, 1986)

- “An intervening variable (mediator) transmits the effect of an independent variable to a dependent variable.” (p. 83, MacKinnon, Lockwood, Hoffman, West & Sheets, 2002)

- a.k.a. Intervening Variable
Conceptual Diagram for Mediator

Independent Variable (X) → Mediator (I) → Dependent Variable (Y)
Causal Steps Approach

Baron & Kenny (1986) analytical approach for mediation

1. X must affect Y (regressing Y on X) \[ \hat{Y} = \beta_0 + \tau X \]

2. X must affect I (regressing I on X) \[ \hat{I} = \beta_0' + \alpha X \]

3. I must affect Y when X is controlled (regressing Y on X and I simultaneously) \[ \hat{Y} = \beta_0'' + \tau' X + \beta I \]

4. the effect of X on Y must be less in (3) than in (1)
Conceptual Diagram for Mediator

Independent Variable (X) → Mediator (I) → Dependent Variable (Y)

α → Mediator (I) → β

τ → τ'
Indirect Effect

- MacKinnon, Warsi & Dwyer (1995): The $\tau - \tau'$ test and the $\alpha \beta$ test are algebraically equivalent.

- Sobel test (1982) for indirect effects ($\alpha \beta$)

$$s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2}$$

- Compare the critical ratio ($ab/s_{ab}$) to appropriate critical value from the Z distribution for a given significant level.
Variations of Sobel test

There are variations for Sobel test

- Goodman (1960): \( s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2 - s_a^2 s_b^2} \)

- Aroian (1944): \( s_{ab} = \sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2} \)

- [http://www.unc.edu/~preacher/sobel/sobel.htm](http://www.unc.edu/~preacher/sobel/sobel.htm)
Mediation = Indirect effect?

- Consistent intervening
- Inconsistent intervening

\[
\begin{align*}
\text{Age (X)} & \rightarrow \text{Safe driving behavior (I)} \\
\text{Safe driving behavior (I)} & \rightarrow \text{Car accident (Y)}
\end{align*}
\]
Multicollinearity in Mediation

- X and I will be correlated

- Effective sample size is approximately $N(1-r^2)$, where $r$ is the correlation between X and I.
Measurement Errors in Mediators

- Using multiple regression for detecting mediation assumes I is measured without errors.
- When measurement error exists but not acknowledged, regression approach leads to underestimation of mediation effect.
- Solution: SEM approach with multiple indicators
Mediation model = Causal model?

- Causal relationship (Shadish, Cook & Campbell, 2002)
  - The cause preceded the effect
  - The cause was related to the effect
  - No plausible alternative explanation for the effect other than the cause
Mediation v.s. confounding effects

- Statistically equivalent models
Distinguish Mediation and Confounding

- Temporal precedence
- Randomized study
- Nature of the variable studied
- Theory

See MacKinnon, Krull & Lockwood (2000) for detail discussions
Kraemer, Wilson, Fairburn and Agras (2002)

- Redefine mediators and moderators in a randomized clinical trials

- Definitions are conceptually similar to Baron and Kenny 1986

- “Treatment moderators specify for whom or under what conditions the treatment works.” (p.878)

- “Treatment mediators identify possible mechanisms through which a treatment might achieve its effects.” (p.878)
Kraemer, Wilson, Fairburn and Agras (2002)

- Emphasizing on temporal order of variables
- Endorse a different analytical approach.
- Using regression model

\[
\hat{Y} = b_0 + b_1 X + b_2 M + b_3 (XM)
\]

for both mediator and moderator
Moderator in Kraemer et. al. (2002)

- Moderator must be a baseline or pre-randomization characteristics.

- Moderator is uncorrelated with treatment.

- Moderator has an interactive effect with treatment on the outcome.
Mediator in Kraemer et. al. (2002)

- Mediator is a measure of event or change occurring during treatment.

- Mediator must correlate with treatment

- Mediator has either a main or an interactive effect on the outcome
Kraemer, Wilson, Fairburn and Agras (2002)

- See handout
Mediators and Moderators in Research Design

- Direct manipulation of mediators in an experimental design to establish causality.
- Use moderators as stratified variables in sampling for following research.
- Moderator effects might suggest potential mediators to be tested.
  - Instruction XYZ worked better for low SES students than for high SES students. Why?